

An Extended LOTOS for the design of Real-Time Systems

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1. Introduction

We give in the following a brief presentation of ET-LOTOS [Lél 95a, Lél 95b]. ET-LOTOS extends with quantitative time the formal description technique LOTOS [ISO 8807]. Other proposals for a "time extended" LOTOS exist. Let us mention [QMF 94] and [BLT 94]. ET-LOTOS serves as basis for the time extension part of E-LOTOS, the new standard for LOTOS currently developed within ISO (ISO/IEC JTC1/SC21).

We assume in the sequel that the reader has a basic knowledge of the syntax and the semantics of LOTOS.

2. Formal semantics and properties of ET-LOTOS

2.1. Datatypes and time domain

In ET-LOTOS, like in LOTOS, datatypes are described in the Abstract Datatype language ACT ONE, that has an initial semantics.

The time domain, denoted D , is defined as the set of values of a given data sort `time` ($D = Q(\text{time})$). Its definition is left free to the will of the specifier provided that the following elements be defined.

- A total order relation represented by " $>$ ".
- An element $0 \in D$ such that: $\forall r \in D: r \neq 0 \Rightarrow r > 0$
- An element $\infty \in D$ such that: $\forall r \in D: r \neq \infty \Rightarrow \infty > r$
- A commutative and associative operation " $+$: $D, D \rightarrow D$ " such that:
 - $\forall r, r1 \in D: r > r1 \Leftrightarrow \exists r' > 0 \bullet (r' + r1) = r$
 - $\forall r, r1 \in D: r > 0 \text{ and } r1 \neq \infty \Rightarrow r + r1 > r1$
 - $\forall r \in D: r + 0 = r$
 - $\forall r \in D: r + \infty = \infty$

The relations " \leq ", and " $-$ " can be derived easily as follows :

- $\forall r, r1 \in D \bullet r \leq r1 \Leftrightarrow (r < r1 \vee r1 = r)$
- $\forall r, r1, r2 \in D \bullet r1 \leq r \Rightarrow (r - r1 = r2 \Leftrightarrow r1 + r2 = r)$
- $\forall r, r1 \in D \bullet r \leq r1 \Rightarrow r - r1 = 0$

In particular, the time domain can be dense as well as discrete, but to be able to give the operational semantics of ET-LOTOS in terms of Labelled Transition Systems (LTS), it must be countable, such as the rational numbers.

2.2 Notations

The following notations hold for the remainder of the paper. G denotes the countable set of common observable gates. $L = G \cup \{\delta\}$ denotes the alphabet of observable gates where δ is the special action denoting successful termination ($\delta \notin G$). δ does not appear explicitly in the syntax of LOTOS. S denotes the set of sorts, V denotes the set of ground terms in the initial algebra associated with the ACT ONE specification: $V = \bigcup_s Q(s)$. $CL = L \times V^*$ denotes the set of observable actions. $A = CL \cup \{i\}$ denotes the alphabet of actions, where the symbol i is reserved for the unobservable internal action ($i \notin L$). g (resp. a) denotes an element of G (resp. A): $g \in G$, $a \in A$. $gv_1 \dots v_n$ and $\delta v_1 \dots v_n$ denote elements of CL , with the v_i 's $\in V$. Capital Greek letters such as Γ will be used to denote subsets of G . D denotes the countable time domain which is the alphabet of time actions. $D_{0\infty} = D - \{0, \infty\}$.

2.3 Syntax of the behaviour part of ET-LOTOS

The collection of ET-LOTOS behaviour expressions is defined by the following BNF expressions. In these expressions, \tilde{x} represents a vector of process names, SP is a selection predicate, the e_i 's represent a term¹ tx , the o_i 's represent either $?x:s$ (with x a variable of sort s) or $!tx$ (with tx a ground term), the x_i 's (resp. tx_i 's) are variables (resp. ground terms) of sorts s_i 's, $d \in D$ and in $@t$, t is a variable of sort time. The new features are printed in italics:

$P ::= Q \text{ where } \tilde{x} := \tilde{Q}$ ²

$Q ::= \text{stop} \mid \text{exit}(e_1, \dots, e_n)\{d\} \mid go_1 \dots o_n @t[SP];Q \mid i@t\{d\};Q \mid \Delta^d Q \mid Q[]Q \mid Q[\Gamma]Q \mid$
 $\text{hide } \Gamma \text{ in } Q \mid Q >> \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \mid Q[>Q \mid x \mid [SP] \rightarrow Q \mid$
 $\text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } Q \mid \text{choice } x_1:s_1, \dots, x_n:s_n [] Q \mid \text{inf} \mid \mid \mid P$

Remark: in $go_1 \dots o_n @t[SP];Q$ we let both $@t$ and $[SP]$ be optional, and use the convention that, if omitted, $[SP] = [\text{true}]$. In $i@t\{d\};Q$, both $@t$ and $\{d\}$ are optional. If omitted, $d = 0$. Similarly $\{d\}$ is optional in $\text{exit}\{d\}$, and exit means implicitly $\text{exit}\{\infty\}$.

The binding powers of the operators are like in LOTOS. For the new operators, Δ^d has the same power as action-prefix and $\text{inf} \mid \mid \mid$ the same as choice $x_1:s_1, \dots, x_n:s_n []$.

An additional shorthand notation: We define the notation $go_1 \dots o_n \{d\};Q$, for $go_1 \dots o_n @t[t \leq d];Q$, provided that t be fresh in Q . Under the same restriction, we also introduce the notation

¹ This term can be: 'any s ' (with $s \in S$)

² For convenience, we suppose, without lack of generality, that there is a single where-clause that gathers all the process declarations of the specification.

$g \circ_1 \dots \circ_n \{d_1, d_2\}; P$ to mean $g @ t[d_1 \leq t \leq d_2]; P$. The meaning of these rewritings will become clear in the next section.

2.4 Semantics of ET-LOTOS

The operational semantics of ET-LOTOS, presented in the following, is of the so-called "time/actions" type. This means that the occurrence of actions and the passing of time are considered as separate concerns, each one being described by a dedicated set of rules.

2.4.1 Notations

P, P', Q, Q' denote ET-LOTOS behaviour expressions.

$P \xrightarrow{a} P'$, with $a \in A$, means that process P may engage in action a and, after doing so, behave like process P' . $P \xrightarrow{g} P'$ means $\exists P', a \bullet P \xrightarrow{a} P' \wedge \text{name}(a) = g$. $P \not\xrightarrow{g}$ means $\neg (P \xrightarrow{g})$ i.e. P cannot perform an action on gate g . $P \xrightarrow{d} P'$, with $d \in D_{0\infty}$, means that process P may idle (i.e. not execute any action in A) during a period of d units of time and, after doing so, behave like process P' . $P \not\xrightarrow{d}$, with $d \in D_{0\infty}$, means that $\nexists P' \bullet P \xrightarrow{d} P'$, i.e. P cannot idle during a period of d units of time. In these expressions, it is required that P and P' be closed, i.e. they do not contain free variables.

2.4.2 Inference rules

In the following inference rules, $d \in D_{0\infty}$, $d_1 \in D$, $d' \in D_\infty$, $g \in G$ and $a \in A$.

We introduce a process, denoted `block`, which has no axiom and no inference rules. This process cannot perform any action and blocks the progression of time.

Inaction

$$(S) \quad \text{stop} \xrightarrow{d} \text{stop}$$

Remark that `stop` cannot perform any action but can idle.

Exit

$$\begin{aligned} (Ex1) \quad & \text{exit}(e_1, \dots, e_n) \{d_1\} \xrightarrow{\delta v_1 \dots v_n} \text{stop} \\ & \text{where } v_i = [t_i] \quad \text{if } e_i = t_i \text{ (a ground term)} \\ & \quad v_i \in Q(s_i) = \{[t] \mid t \text{ is a ground term of sort } s_i\} \quad \text{if } e_i = \text{any } s_i \\ (Ex2) \quad & \text{exit}(e_1, \dots, e_n) \{d_1 + d\} \xrightarrow{d} \text{exit}(e_1, \dots, e_n) \{d_1\} \\ (Ex3) \quad & \text{exit}(e_1, \dots, e_n) \{d_1\} \xrightarrow{d} \text{stop} \quad (d > d_1) \end{aligned}$$

The $\{d_1\}$ attribute is called the life reducer. Its role is to restrict the time period during which the process can terminate successfully: `exit` $\{d_1\}$ can only perform δ during the next d_1 time units. If `exit` $\{d_1\}$ has not performed δ yet after d_1 time units, it is too late and the process turns into `stop` (rule Ex3).

Observable action-prefix

$$\begin{aligned}
 \text{(AP1)} \quad & go_1 \dots o_n @t[SP];P \xrightarrow{gv_1 \dots v_n} [v_1/o_1, \dots v_m/o_m, 0/t]P \\
 & \text{if } \vdash [v_1/o_1, \dots v_m/o_m, 0/t]SP \\
 & \quad v_i = [w] \quad \text{if } o_i = !w \\
 & \quad v_i \in Q(s) = \{[w] \mid w \text{ is a ground term of sort } s\} \quad \text{if } o_i = ?x:s \\
 & \text{and where } v_i/o_i = v_i/x \quad \text{if } o_i = ?x:s \\
 & \quad v_i/o_i \text{ is void} \quad \text{if } o_i = !w \\
 \text{(AP2)} \quad & go_1 \dots o_n @t[SP];P \xrightarrow{d} go_1 \dots o_n @t[[t+d/t]SP];[t+d/t]P
 \end{aligned}$$

In $@t$, t is a variable of sort time . This variable is used to measure the delay actions were being offered on g when one occurred. When an action occurs (rule AP1), t is instantiated. Instantiating t by 0 is logical: $go_1 \dots o_n @t[SP];P$ describes a process at a given instant and the counting of t starts at that instant. So, t is still at 0 if the process immediately does an action on gate g . The way the value of t is kept up to date if $go_1 \dots o_n @t[SP];P$ idles is defined by AP2.

The t variable can appear in the selection predicate SP , if there is one. The conditions joined with AP1 express that the only possible instantiations for the attributes of g are the ones that make SP true at that instant.

Internal action-prefix

$$\begin{aligned}
 \text{(I1)} \quad & i@t\{d1\};P \xrightarrow{i} [0/t]P \\
 \text{(I2)} \quad & i@t\{d1+d\};P \xrightarrow{d} i@t\{d1\};[t+d/t]P
 \end{aligned}$$

There is no rule like Ex3 for the internal action-prefix. $i@t\{d1\};P$ cannot idle more than $d1$ time units. If it reaches this limit, time is blocked. The only solution left is to accomplish i . This means that, in Timed Extended LOTOS, the occurrence of i is compulsory. The semantics of $i@t\{d1\};P$ is that i *shall* occur during the next $d1$ time units³. On the other hand, the semantics of $exit(d1)$ is that δ *may* occur within the next $d1$ time units.

Delay prefixing

$$\begin{aligned}
 \text{(D1)} \quad & \frac{P \xrightarrow{a} P'}{\Delta^0 P \xrightarrow{a} P'} \\
 \text{(D2)} \quad & \Delta^{d1+d} P \xrightarrow{d} \Delta^{d1} P \\
 \text{(D3)} \quad & \frac{P \xrightarrow{d} P'}{\Delta^{d1} P \xrightarrow{d+d1} P'}
 \end{aligned}$$

$\Delta^d;P$ expresses that P will be delayed by d time units.

Choice

$$\begin{aligned}
 \text{(Ch1)} \quad & \frac{P \xrightarrow{a} P'}{P[]Q \xrightarrow{a} P'} & \text{(Ch1')} \quad & \frac{Q \xrightarrow{a} Q'}{P[]Q \xrightarrow{a} Q'} & \text{(Ch2)} \quad & \frac{P \xrightarrow{d} P', Q \xrightarrow{d} Q'}{P[]Q \xrightarrow{d} P'[]Q'}
 \end{aligned}$$

Remark rule Ch2: the passing of time does not resolve a choice. Rule Ch2 also states that both operands evolve in time at the same pace.

³ Of course, in a choice context, the occurrence of i could be prevented by another offered action.

Generalized choice

The semantics of choice $x_1:s_1, \dots, x_n:s_n[]P$ is defined via an auxiliary operator, denoted $\text{Achoice}(d)$ $x_1:s_1, \dots, x_n:s_n[]P$, where $d \in D_\infty$. Achoice stands for *AgedChoice*. By definition, choice $x_1:s_1, \dots, x_n:s_n[]P = \text{Achoice}(0) \ x_1:s_1, \dots, x_n:s_n[]P$.

$$\begin{aligned}
 \text{(GC1)} \quad & \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{a} P'}{\text{Achoice}(0) \ x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{a} P'} \\
 \text{(GC2)} \quad & \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{d} P'', \ P'' \xrightarrow{a} P'}{\text{Achoice}(d) \ x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{a} P'} \quad \text{if } d > 0
 \end{aligned}$$

where the tx_i are ground terms with $[tx_i] \in Q(s_i)$

$$\text{(GC3)} \quad \frac{[tx_1/x_1, \dots, tx_n/x_n]P \xrightarrow{d+d'} \quad \forall \langle tx_1, \dots, tx_n \rangle \bullet [tx_i] \in Q(s_i), \ i = 1, \dots, n}{\text{Achoice}(d') \ x_1:s_1, \dots, x_n:s_n[]P \xrightarrow{d} \text{Achoice}(d+d') \ x_1:s_1, \dots, x_n:s_n[]P}$$

Parallel composition

$$\begin{aligned}
 \text{(PC1)} \quad & \frac{P \xrightarrow{a} P'}{P \mid [\Gamma] \mid Q \xrightarrow{a} P' \mid [\Gamma] \mid Q} \quad (\text{name}(a) \notin \Gamma \cup \{\delta\}) & \text{(PC3)} \quad \frac{P \xrightarrow{d} P', \ Q \xrightarrow{d} Q'}{P \mid [\Gamma] \mid Q \xrightarrow{d} P' \mid [\Gamma] \mid Q'} \\
 \text{(PC1')} \quad & \frac{Q \xrightarrow{a} Q'}{P \mid [\Gamma] \mid Q \xrightarrow{a} P \mid [\Gamma] \mid Q'} \quad (\text{name}(a) \notin \Gamma \cup \{\delta\}) \\
 \text{(PC2)} \quad & \frac{P \xrightarrow{a} P', \ Q \xrightarrow{a} Q'}{P \mid [\Gamma] \mid Q \xrightarrow{a} P' \mid [\Gamma] \mid Q'} \quad (\text{name}(a) \in \Gamma \cup \{\delta\})
 \end{aligned}$$

Infinite parallel composition

$$\begin{aligned}
 \text{(IP1)} \quad & \frac{P \xrightarrow{a} P'}{\inf ||| P \xrightarrow{a} P' \quad ||| \ (\inf ||| P)} & \text{(IP2)} \quad \frac{P \xrightarrow{d} P'}{\inf ||| P \xrightarrow{d} \inf ||| P'}
 \end{aligned}$$

$\inf ||| P$ corresponds to an infinity of occurrences of P evolving in parallel. In ET-LOTOS, such a behaviour cannot be described by a recursive process like $P_s := P ||| P_s$, because unguarded recursions block time (see [LéL 95b]).

Hide

$$\begin{aligned}
 \text{(H1)} \quad & \frac{P \xrightarrow{a} P'}{\text{hide } \Gamma \text{ in } P \xrightarrow{a} \text{hide } \Gamma \text{ in } P'} \quad (a \notin \Gamma) \\
 \text{(H2)} \quad & \frac{P \xrightarrow{a} P'}{\text{hide } \Gamma \text{ in } P \xrightarrow{i} \text{hide } \Gamma \text{ in } P'} \quad (a \in \Gamma) \\
 \text{(H3)} \quad & \frac{P \xrightarrow{d} P', \ \forall g \in \Gamma \bullet (P \xrightarrow{g} \wedge \forall P'' \ \forall d' < d \bullet (P \xrightarrow{d'} P'' \Rightarrow P'' \xrightarrow{g} P))}{\text{hide } \Gamma \text{ in } P \xrightarrow{d} \text{hide } \Gamma \text{ in } P'}
 \end{aligned}$$

Rule (H3) expresses the *maximal progress* principle adopted for ET-LOTOS. This principle states that the hidden events must occur as soon as possible. So, the process can only idle if no hidden action is possible.

Enabling

$$\text{(En1)} \quad \frac{P \xrightarrow{a} P'}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{a} P' \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q} \quad (\text{name}\{a\} \neq \delta)$$

$$\begin{aligned}
 (\text{En2}) \quad & \frac{P \xrightarrow{\delta v_1 \dots v_n} P'}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{i} [v_1/x_1, \dots, v_n/x_n]Q} \quad \forall j \leq n \bullet v_j \in Q(s_j) \\
 (\text{En3}) \quad & \frac{P \xrightarrow{d} P', P \not\xrightarrow{\delta}, \forall P'' \forall d' < d \bullet (P \xrightarrow{d'} P'' \Rightarrow P'' \not\xrightarrow{\delta})}{P \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q \xrightarrow{d} P' \gg \text{accept } x_1:s_1, \dots, x_n:s_n \text{ in } Q}
 \end{aligned}$$

The occurrence of δ is hidden by the enabling operator. According to the maximal progress principle, it must occur as soon as possible.

Disabling

$$\begin{aligned}
 (\text{Di1}) \quad & \frac{P \xrightarrow{a} P'}{P[>Q \xrightarrow{a} P' [>Q]} \quad (\text{name}(a) \neq \delta) & (\text{Di2}) \quad & \frac{Q \xrightarrow{a} Q'}{P[>Q \xrightarrow{a} Q'} \\
 (\text{Di3}) \quad & \frac{P \xrightarrow{a} P'}{P[>Q \xrightarrow{a} P'} \quad (\text{name}(a) = \delta) & (\text{Di4}) \quad & \frac{P \xrightarrow{d} P', Q \xrightarrow{d} Q'}{P[>Q \xrightarrow{d} P' [>Q'}
 \end{aligned}$$

Guard

$$\begin{aligned}
 (\text{G1}) \quad & \frac{P \xrightarrow{a} P'}{[SP] \rightarrow P \xrightarrow{a} P'} \quad \text{if } DS \vdash SP & (\text{G2}) \quad & \frac{P \xrightarrow{d} P'}{[SP] \rightarrow P \xrightarrow{d} P'} \quad \text{if } DS \vdash SP \\
 (\text{G3}) \quad & [SP] \rightarrow P \xrightarrow{d} \text{stop} \quad \text{if } \neg DS \vdash SP
 \end{aligned}$$

Let

$$\begin{aligned}
 (\text{L1}) \quad & \frac{[tx_1/x_1, \dots, tx_n/x_n] P \xrightarrow{a} P'}{\text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } P \xrightarrow{a} P'} & (\text{L2}) \quad & \frac{[tx_1/x_1, \dots, tx_n/x_n] P \xrightarrow{d} P'}{\text{let } x_1=tx_1, \dots, x_n=tx_n \text{ in } P \xrightarrow{d} P'}
 \end{aligned}$$

Process instantiation

$$\begin{aligned}
 (\text{In1}) \quad & \frac{[g_1/h_1, \dots, g_n/h_n] P \xrightarrow{a} P', Q[h_1, \dots, h_n] := P}{Q[g_1, \dots, g_n] \xrightarrow{a} P'} & (\text{In2}) \quad & \frac{[g_1/h_1, \dots, g_n/h_n] P \xrightarrow{d} P', Q[h_1, \dots, h_n] := P}{Q[g_1, \dots, g_n] \xrightarrow{d} P'}
 \end{aligned}$$

Let us outline some interesting features of the semantic rules defined above:

- The LOTOS rules are kept unchanged.
- The alphabet A of actions is kept as is (e.g. no additional time stamps in action labels). It is just extended with time actions from a separate set D .

2.5. Properties

ET-LOTOS exhibits many interesting properties (the proofs can be found in [LéL 95b]):

- The operational semantics of ET-LOTOS is consistent.
- Time transitions are deterministic: $\forall P \bullet (P \xrightarrow{d} P' \wedge P \xrightarrow{d} P'') \Rightarrow P' = P''$.
- Time transitions are closed under the relation \leq : $P \xrightarrow{d} \Rightarrow \forall d' \in]0, d[\bullet P \xrightarrow{d'}$.
Furthermore, $P \xrightarrow{d} P' \Rightarrow \forall d' \in]0, d[\bullet \exists d'' \bullet P \xrightarrow{d'} P'' \xrightarrow{d''} P' \wedge d = d' + d''$.
- Time transitions are additive: $P \xrightarrow{d} P'$ and $P' \xrightarrow{d'} P''$ implies $P \xrightarrow{d+d'} P''$.
- Strong bisimulation \sim is a congruence.
- ET-LOTOS is upward compatible with LOTOS, according to the definition given in [NiS92], but for guarded specifications only.

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